

2-D Wavelet-Based Spectra Analysis for the Classification of Geophysical Images¹

Analisi dello Spettro Wavelet 2D per la classificazione di Immagini Geofisiche

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Riassunto: In questo lavoro proponiamo un metodo basato sullo spettro wavelet bi-dimensionale per la stima del parametro (direzionale) di Hurst in campi Gaussiani frazionari isotropici ed anisotropici. Tale metodo può essere applicato a immagini auto-similari e, in generale, a dati d -dimensionali. Nella parte applicativa consideriamo la classificazione delle immagini satellitari delle nuvole/temperatura. In particolare, vengono prima stimati i coefficienti di Hurst direzionali e successivamente utilizzati come input delle procedure standard di *machine learning*.

Keywords: Scaling, Wavelets, Self-similarity, 2D wavelet spectra.

1. Introduction

The aim of the paper is to propose a wavelet-based spectra method to explore geophysical images and d -dimensional data. The definition of such spectra is quite natural and a few examples of its use are scattered in the literature (Heneghan *et al.* (1996); Parra *et al.*, 2003). However, formal definition and systematic analysis of such spectra is not provided. In addition, the existing examples of use mainly utilize isotropic images. Since in the tensor-product wavelet multiresolution analysis of d dimensional data $2^d - 1$ detail spaces are possible, where each space contains the hierarchy of subspaces with nested dyadic resolutions, it is quite natural to assess the energy scaling in each hierarchy. This leads to $2^d - 1$ different power spectra describing a single d -dimensional data set. For example, multiresolution analysis of images leads to the detail spaces that are described as “horizontal”, “vertical” or “diagonal,” depending on the selection of the decomposing 2-D wavelet, or equivalently, the order of applications of high- and low-pass wavelet filters on the rows and columns of 2-D objects. Each of the three directional details contains a

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nested hierarchy of submatrices corresponding to image details at different scales and each leads to a distinctive power spectra.

The 2-D fractional Brownian motion (fBm) process, $B_H(\mathbf{u})$ is a standard model for self-similar random fields. For this process the variance of the wavelet coefficients are given by

$$E \left[\left| d_{j,\mathbf{k}}^i \right|^2 \right] = \frac{\sigma_H^2}{2} V_{\psi^i} 2^{-(2H+2)j}, \quad (1)$$

where

$$V_{\psi^i} = \iint \psi^i(\mathbf{p} + \mathbf{q}) \cdot \psi^i(\mathbf{q}) |\mathbf{p}|^{2H} d\mathbf{p} d\mathbf{q} \quad (2)$$

depends only on wavelets ψ^i and H , but not on the scale j . (The derivation is given in the Appendix I of Nicolis *et al.* (2007)).

The Hurst coefficients of a 2D fBm field can be estimated from the slope in the linear equation resulting from the logarithm transformation of Eq. (1).

In the application we illustrate how the wavelet-based estimators of directional Hurst exponent can be utilized in classification of satellite images. The source of the data is EU-METSAT (<http://www.eumetsat.int>). In order to assess the efficacy of Hurst exponents to discriminate images belonging to groups with different texture characteristic, we performed the following experiment. We considered a data subset of 160 satellite images of the Gulf of Guinea (West cost of Africa and South Atlantic Ocean). The images are taken at 40 consecutive days (11/1/2006 - 12/10/2006) at midnight, in the morning, at noon and in the evening. Then, we divided the test set in a training set and a validation set. There are four distinctive groups the images are belonging to. We trained a linear discriminator based on the Hurst exponents on a training set, and then we used the developed discriminatory rule to classify each image from the validation set to one of the four groups. The linear discrimination procedure fitted a multivariate normal density to each group, with a pooled estimate of covariance. We used the images taken on odd days as training set, while the validation set was composed from the images taken on even days. Significant discrimination occurred by using horizontal and vertical Hurst exponents of the images taken at noon from the images taken at other times (evening, night, morning). From 80 images belonging to the validation set, 73 have been correctly classified, achieving correct classification rate of 91.25%.

References

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